BFKL at next-to-leading order

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Abstract

This is a summary of the contributions on the next-to-leading order corrections to the BFKL equation which were presented to the 'Small-x and Diffraction' working group at the 1998 Durham Workshop on HERA Physics.

The original BFKL equation [1] allowed the computation of elastic scattering amplitudes in the Regge limit to leading logarithmic (LO) accuracy, i.e. all terms $\sim (\alpha_s \ln s)^n$ are summed. The key element is the gluon Green function:

$$F(s,k) = \int \frac{d\gamma}{2\pi i} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \frac{1}{\omega - \bar{\alpha}_s \chi(\gamma)} \left(\frac{k_1^2}{k_2^2}\right)^{\gamma}. \tag{1}$$

The scale s_0 is arbitrary, $\bar{\alpha}_s = N_c \alpha_s / \pi$ is fixed and, at leading order,

$$\chi(\gamma) = \chi_0(\gamma) = -2\gamma_E - \psi(\gamma) - \psi(1-\gamma). \tag{2}$$

The $s \to \infty$ behaviour of the amplitude is governed by the right-most ω-plane singularity which occurs at $\gamma = \bar{\gamma}$ where

$$\omega = \bar{\alpha}_s \chi(\bar{\gamma}). \tag{3}$$

The conformal invariance of the LO kernel allows us to write equation (1). The NLO corrections [2] break the conformal invariance by leading to the running of the QCD coupling. However, it is possible

to study the conformally invariant part of the NLO equation. In this case, equation (1) is still relevant but now the kernel is

$$\chi(\gamma) = \chi_0(\gamma) + \bar{\alpha}_s \chi_1(\gamma). \tag{4}$$

The NLO kernel depends upon the scale s_0 in a straightforward way ($s_0 = k_1 k_2$ is usually chosen). The NLO corrections contained in χ_1 are huge for typical values of α_s which is a cause for serious concern since we are attempting a perturbative expansion [2, 3, 4]. For example, at $\gamma = 1/2$

$$\chi(\gamma) = \omega_0 (1 - 6.5 \ \bar{\alpha}_s)$$

where $\omega_0 = \bar{\alpha}_s \chi_0(1/2) = 4 \ln 2 \ \bar{\alpha}_s$ is the leading eigenvalue of the LO BFKL kernel.

Marcello Ciafaloni and Gavin Salam pointed out that the problem is alleviated by identifying and resumming a large part of the NLO corrections: that with double logarithms in the transverse momenta (corresponding to the $1/\gamma^3$ and $1/(1-\gamma)^3$ parts of the NLO kernel). After ensuring the consistency of these double logarithms to all orders, the perturbation series is much more convergent.

The double logarithms are closely associated with the choice of scale s_0. Schematically one can see that they arise through a change of scale s_0 from k_1^2 to $k_1 k_2$. In the situation with $k_1 \gg k_2$, the quantity which exponentiates is roughly

$$\bar{\alpha}_s \ln\left(\frac{s}{k_1^2}\right) \ln\left(\frac{k_1^2}{k_2^2}\right) = \bar{\alpha}_s \ln\left(\frac{s}{k_1 k_2}\right) \ln\left(\frac{k_1^2}{k_2^2}\right) - \frac{1}{2}\bar{\alpha}_s \ln^2\left(\frac{k_1^2}{k_2^2}\right) .$$

If one writes one's expansion in terms of scale $s_0 = k_1 k_2$ (i.e. the RHS) the exponentiation leads not just to a large NLO term, but also to a whole series of NNLO, NNNLO, ... terms with double transverse logarithms, which can be large. These should therefore be resummed. At first sight the simplest way to do so seems be to rewrite one's expansion in terms of $s_0 = k_1^2$ (LHS). The problem is that such a scale choice spoils the convergence (again by double logarithms) of the kernel in the case when $k_1 \ll k_2$, where the 'natural' (DGLAP) scale choice is instead $s_0 = k_2^2$.

A way of treating both regions on an equal footing while correctly resumming the double logarithms was proposed in [5] and is related to an approach suggested by the Lund group [6]. Figure 1 shows that such a resummation greatly improves the convergence of the kernel. The various schemes correspond to different treatments of less singular terms (in particular those related to collinear branching and the running coupling), which remain to be accounted for in such a way as to ensure consistency with the renormalisation group (see below, and also [7]).

The phenomenological importance of matching to the known DGLAP limit and of including kinematical and other 'finite z' effects, e.g. coherence, have been and will continue to be explored by a number of groups. For example, see the contribution of Kharrazhia and Stasto in these proceedings [8]. Lynn Orr showed us the crucial importance of imposing the correct kinematics. She focussed on the azimuthal decorrelation in dijet production at the Tevatron where the data were compared with results from a BFKL Monte Carlo with the correct kinematics imposed [9].

So far we ignored the fact that the QCD coupling runs. At NLO we need to account for the breaking of conformal invariance due to running coupling effects. The eigenfunctions of the kernel are no longer simple powers of the momenta and a more sophisticated approach is required. Since the BFKL formalism integrates over all momenta, the running of the coupling invariably leads to
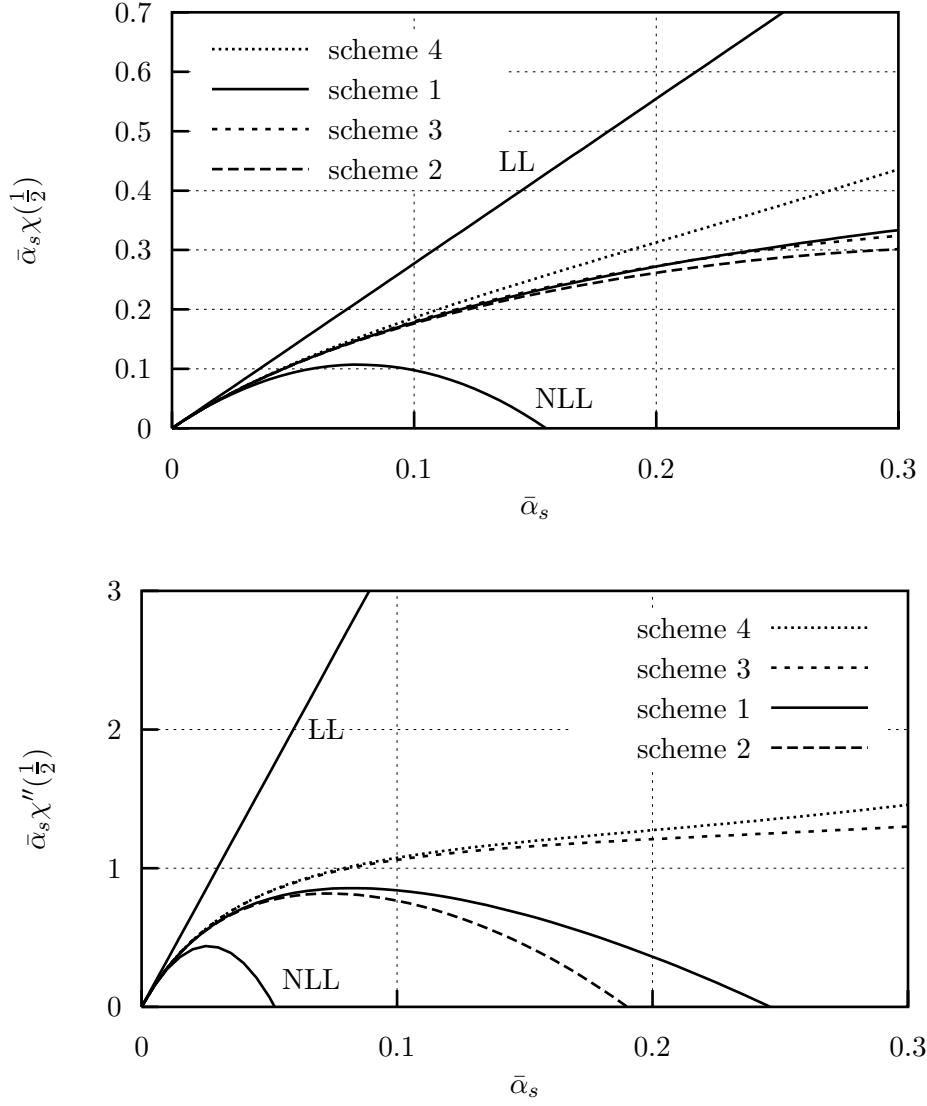
Figure 1: The result of resummation on χ and its second derivative; shown for $n_f = 0$. A symmetric version of the pure NLO kernel has been used to redefine $k^{2(\gamma-1)}$ as $\sqrt{\alpha_s(k^2)}k^{2(\gamma-1)}$ in the eigenfunctions.

divergences associated with the failure of QCD perturbation theory in the infrared. To regularise the theory, one might take a phenomenological approach and introduce by hand some regulator, e.g. a gluon mass to freeze the QCD coupling at some low scale. This has been done before, in the case of the LO BFKL equation [10]. However, in single scale problems like deep inelastic scattering at small x one does not need to speculate. The collinear factorisation of deep inelastic cross-sections allows the ill behaved infrared dynamics to be factorised into parton distribution functions and it is their scale dependence that is predicted by renormalisation group (RG) (i.e. DGLAP) equations of QCD. Ciafaloni stressed the importance of working within a RG consistent framework.

A systematic approach to small-x structure functions was presented by Robert Thorne. A few years ago Catani showed how to write the DGLAP equations in a manifestly factorisation scheme independent way [11]:

$$\frac{\partial F_i}{\partial \ln Q^2} = \Gamma_{ij} F_j \tag{5}$$

where $i, j = 2, L$ and Γ_{ij} is the physical anomalous dimension matrix. The NLO corrections to the BFKL equation can be used to determine the scale dependence of the structure functions F_2 and F_L.[1] In particular, the physical anomalous dimension matrix is computed including the complete two-loop DGLAP splitting functions supplemented with the leading twist information coming from the NLO BFKL equation; corrections to this are strictly $\mathcal{O}(\alpha_s(\mu))$. The only ambiguity in this procedure is in the choice of the renormalisation scale μ. Thorne pointed out that the 'natural' choice $\mu = Q$ has disasterous consequences, as can be seen in the upper of Fig.2: the dotted line is the evolution of the longitudinal structure function at low x with $\mu = Q$ at NLO, as compared to the LO evolution shown by the solid line (in both cases $F_L(x, Q^2)$ is parameterized as $(0.1/x)^{-0.3}$ at $\ln(Q^2/\Lambda^2) = 8$). Thorne adopted the BLM scale fixing procedure [12] which, for some observable

$$R(Q) = A\alpha_s(\mu) - \beta_0\alpha_s(\mu)^2[A\ln(Q/\mu) + B] + \alpha_s(\mu)^2 C + \cdots,$$

states that the appropriate scale is determined by ensuring that

$$A\ln(Q/\mu) + B = 0. \tag{6}$$

In this case A and B depend upon α_s and x, and the scale choice corresponds to

$$\alpha_s(\mu) \approx \frac{1}{\beta_0(\ln(Q^2/\Lambda^2) + 3.6\alpha_s(Q^2)(\ln 1/x)^{1/2})}$$

at small x and for all physical anomalous dimensions (and parton anomalous dimensions in sensible schemes). This scale choice has the effect of absorbing the large higher order corrections into the running of the coupling, i.e. subsequent corrections are small and support the use of perturbation theory, as shown in the lower of Fig.2. It also has the peculiar effect of reducing the coupling at low x. The physical origin of such strange behaviour was the cause for some discussion, though it now seems to be due to infrared diffusion in the BFKL equation only affecting the inputs for structure functions, while the evolution is sensitive to ultraviolet diffusion, and hence scales larger than Q^2 as x decreases [13]. Thorne also fixed the BLM scale for Γ_{22} ($\sim P_{qq}$) at high x, and showed us the result of fitting all the DIS data fitted by the MRST group using lowest order evolution with the scale fixing. The

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[1]Actually we don't have all the information yet since the coupling of the gluons to the photon via a quark loop has not yet been computed at NLO.

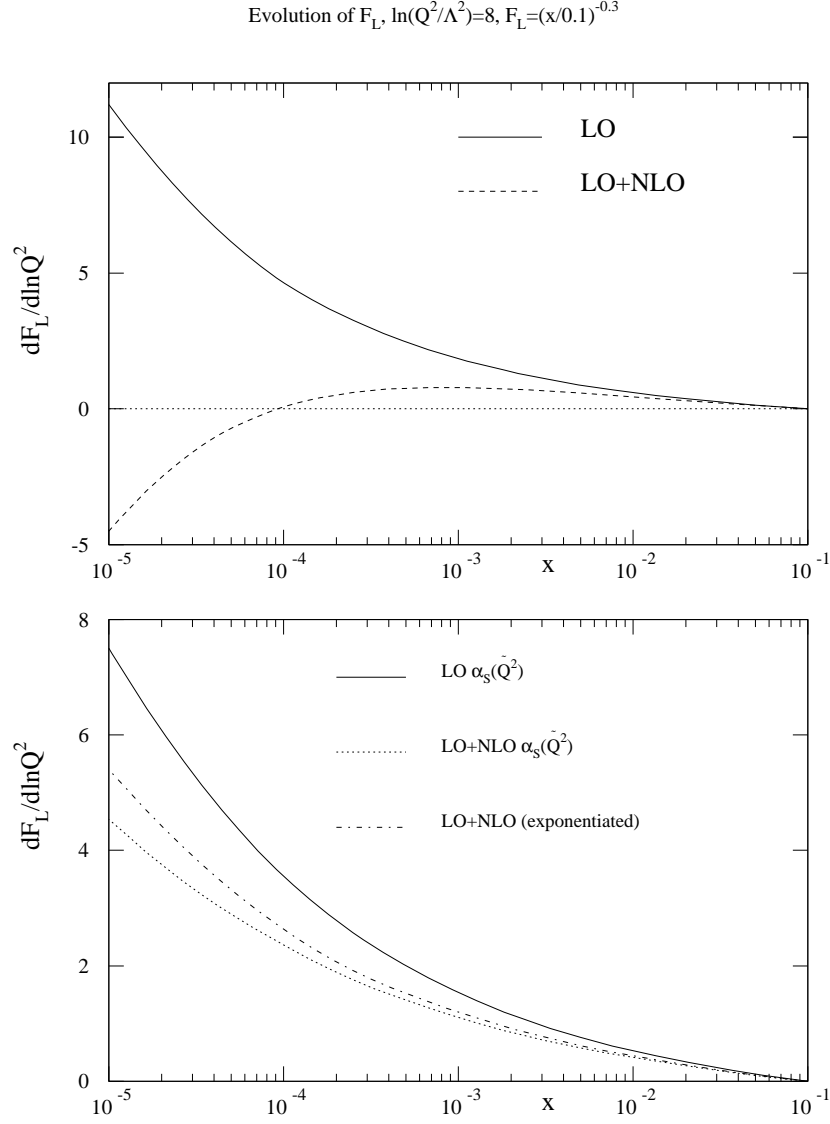Evolution of F_L, $\ln(Q^2/\Lambda^2)=8$, $F_L=(x/0.1)^{-0.3}$

Figure 2: Evolution of F_L at LO and NLO.

resulting fit produces a remarkably small chi squared, as can be seen in Table 1 which compares the latest MRST fit with the lowest order BLM fit (LO(x)). It also leads to $F_L(x, Q^2)$ following the shape of $F_2(x, Q^2)$ at low Q^2, unlike the conventional approaches.

Data set	No. of data pts	LO(x)	MRST
H1 ep	221	149	164
ZEUS ep	204	246	270
BCDMS μp	174	241	249
NMC μp	130	118	141
NMC μd	130	81	101
SLAC ep	70	87	119
E665 μp	53	59	58
E665 μd	53	61	61
CCFR $F_2^{\nu N}$	66	57	93
CCFR $F_3^{\nu N}$	66	65	68
NMC n/p	163	176	187
total	1330	1339	1511

Table 1: Comparison of χ^2 for fits using the full leading order (including $\ln(1/x)$ terms) renormalization scheme consistent expression with BLM scale, and the two-loop MRST fit.

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